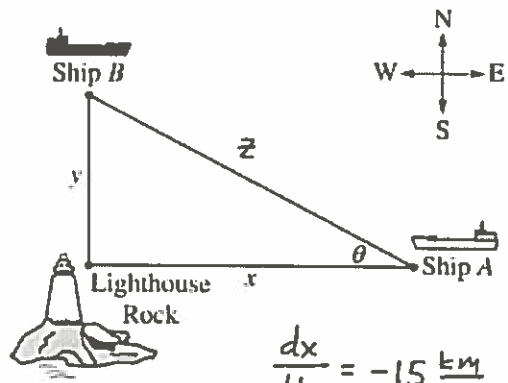
KEY

(2002B AB6) Ship A is traveling due west toward Lighthouse Rock at a speed of 15 kilometers per hour (km/hr). Ship B is traveling due north away from Lighthouse Rock at a speed of 10 km/hr. Let x be the distance between Ship A and Lighthouse Rock at time t, and let y be the distance between Ship B and Lighthouse Rock at time t, as shown in the figure at right.



$\frac{dx}{dt} = -15 \frac{km}{hr}$

$\frac{dy}{dt} = 10$ km/hr

(a) Find the distance, in kilometers, between Ship A and Ship B when $x = 4$ km and $y = 3$ km.

$z = 5$ km

(b) Find the rate of change, in km/hr, of the distance between the two ships when $x = 4$ km and $y = 3$ km.

$x^2 + y^2 = z^2$

$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}$

$4(-15) + 3(10) = 5 \frac{dz}{dt}$

$\frac{dz}{dt} = -6$ km/hr

(c) Let θ be the angle shown in the figure. Find the rate of change of θ, in radians per hour, when $x = 4$ km and $y = 3$ km.

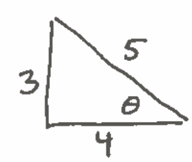
$\tan \theta = \frac{y}{x}$

$\sec^2\theta \frac{d\theta}{dt} = \frac{x \frac{dy}{dt} - y \frac{dx}{dt}}{x^2}$

$\left(\frac{5}{4}\right)^2 \frac{d\theta}{dt} = \frac{4(10) - 3(-15)}{16}$

$\frac{d\theta}{dt} = \frac{85}{16} \cdot \frac{16}{25} = \frac{85}{25}$ rad/hr

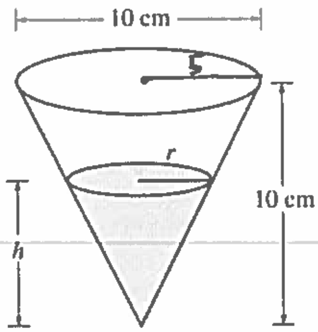
$= \frac{17}{5}$ rad/hr



$\cos\theta = \frac{4}{5}$

$\sec\theta = \frac{5}{4}$

(2002 AB5) A container has the shape of an open right circular cone, as shown in the figure below. The height of the container is 10 cm and the diameter of the opening is 10 cm. Water in the container is evaporating so that its depth h is changing at the constant rate of $-\frac{3}{10}$ cm/hr.



$r = \frac{1}{2}h$

(a) Find the volume V of water in the container when $h = 5$ cm. Indicate units of measure.

$V = \frac{1}{3}\pi r^2 h$

MOMENT

$h = 5$

RATE

$\frac{dh}{dt} = -\frac{3}{10}$ cm/hr

$V = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 h$

$V = \frac{1}{12}\pi h^3$

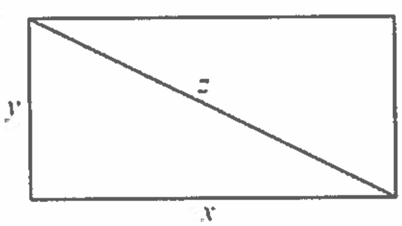
$V = \frac{1}{12}\pi(5)^3 = \frac{125}{12}\pi \text{ cm}^3$

(b) Find the rate of change of the volume of water in the container, with respect to time, when $h = 5$ cm. Indicate units of measure.

$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$

$\frac{dV}{dt} = \frac{1}{4}\pi(5)^2\left(-\frac{3}{10}\right)$

$\frac{dV}{dt} = -\frac{75\pi}{40} = -\frac{15\pi}{8} \text{ cm}^3/\text{hr}$



The sides of the rectangle at right increase in such a way that $\frac{dz}{dt} = 1$ and $\frac{dx}{dt} = 3\frac{dy}{dt}$. At the instant when $x = 4$ and $y = 3$, what is the value of $\frac{dx}{dt}$?

$z = 5$

$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$4\frac{dx}{dt} + 3\left(\frac{1}{3}\frac{dx}{dt}\right) = 5$

$5\frac{dx}{dt} = 5 \qquad \frac{dx}{dt} = 1$